

IONIC EQUILIBRIUM AND THE ELECTRICAL CONDUCTIVITY IN THUNDERCLOUDS

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ABSTRACT

The electrical conductivity existing in electrified clouds is computed under the assumption that a primary positive dipole charge distribution exists within the cloud and that the small ion concentrations are controlled by the rate of ion production from cosmic ray ionization, and by the rates of ion loss from the separate processes of ion recombination, diffusion, and conduction. The results show that the cloud is nearly non-conducting by the small ion conduction mechanism when electric fields are present. The charge transfer mechanisms in the cloud boundary regions and in the cloud updraft are examined, and the thickness of the sheathing layer charge distribution in quasi-static cloud boundaries is estimated.

1. INTRODUCTION

A preceding analysis of the quasi-static cloud system charge distribution stresses the importance of the electrical conductivity [6]. Despite its vital role in thunderstorm theory, the conductivity measurement within thunderclouds presents such difficulties that no valid data have yet been obtained [2]. In the absence of such data, we must resort to basic assumptions regarding the processes of ion generation and ion loss in clouds from which numerical estimates of the conductivity in clouds can be obtained. The following analysis, directed to this purpose, is primarily an analysis of the ion concentrations existing in the cloud particle distribution. The electrical conductivity is related to the ion concentration by known factors, i.e., $\lambda = n_+ e_+ u_+ + n_- e_- u_-$, where n_{\pm} are the ionic concentrations, e_{\pm} the ionic charges, and u_{\pm} the ionic mobilities which vary inversely with the air density. Hence the conductivity is readily evaluated once the ion concentrations are known.

2. IONIC EQUILIBRIUM IN WARM CLOUDS

Within clouds the small ion densities are controlled by the rate of ion production and by the rate of ion loss from the separate processes of ionic recombination, molecular diffusion to the cloud particles, and by the conduction currents to charged and uncharged cloud particles. We will assume the rate of ion formation, q , to be that provided by cosmic rays, varying from values of near $5 \text{ cm}^{-3} \text{ sec}^{-1}$ at the base of the cloud to near $30 \text{ cm}^{-3} \text{ sec}^{-1}$ in the upper cloud levels [8]. The rates of ion loss from the processes of ionic recombination and diffusion acting separately are given by $\alpha n_1 n_2$ and $4\pi a D_1 N n_1$ and $4\pi a D_2 N n_2$, where α is the recombination coefficient for small ions, n_1 and n_2 are the positive and negative small ion concentrations, a is the radius and N the concentration of the cloud particles, and D_1 and D_2 are the ionic diffusion constants. Although the positive and negative diffusion coefficients are recognized as being unequal and of significance to initial charge separation processes in clouds [3], their difference is not of con-

sequence to the present argument and we will assume that $D_1 = D_2 = D$.

The loss of ions by conduction occurs as the result of the flow of ions to the net charges on the cloud drops and to the polarization charges that are induced on the drops by the existing electric fields in storms. The currents to the drop are evaluated [4] to be

$$I_1 = \frac{\pi n_1 e u_1}{3 E a^2} [3 E a^2 - Q]^2 \quad (1a)$$

and

$$I_2 = \frac{\pi n_2 e u_2}{3 E a^2} [3 E a^2 + Q]^2, \quad (1b)$$

where the subscripts 1 and 2 refer to the positive and negative components, respectively, and E is the electric field. Since the total current to the drop is zero when equilibrium is established, we may equate the expressions for the currents to obtain the relation

$$Q = 3 E a^2 \left[\frac{(n_1 u_1 / n_2 u_2)^{1/2} - 1}{(n_1 u_1 / n_2 u_2)^{1/2} + 1} \right], \quad (2)$$

where Q is the free equilibrium charge carried by the drop. Two important variations can occur. When $n_1 u_1 = n_2 u_2$, final equilibrium is achieved when $Q = 0$. When $n_1 u_1 \neq n_2 u_2$, the final droplet equilibrium charge is other than zero. Gunn [4] refers to this process of drop charging in the presence of an electric field and a nonequal ion concentration as "hyperelectrification." The relations hold for all $Q \leq |3 E a^2|$. For $Q > 3 E a^2$, the electric field at the surface of the drop is everywhere repulsive to positive ions and the positive ion current to the drop is zero; for $Q < -3 E a^2$, the same is true for negative ions. To the same approximation determined for the diffusion coefficients, we will assume that $u_1 = u_2 = u$.

If the separate processes of ion loss are independent, then the rate of change of the ion concentrations per cubic centimeter per second is given by summing the separate processes of ion generation and ion loss, whence

$$\frac{dn_1}{dt} = u \operatorname{div} (E n_1) + q - \alpha n_1 n_2 - 4\pi a D N n_1 - \frac{\pi n_1 u N}{3 E a^2} [3 E a^2 - Q]^2 \quad (3a)$$

and

$$\frac{dn_2}{dt} = u \operatorname{div} (\mathbf{E}n_2) + q - \alpha n_1 n_2 - 4\pi a D N n_2 - \frac{\pi n_2 u N}{3Ea^2} [3Ea^2 + Q]^2. \quad (3b)$$

Let us examine briefly the assumption of linear independence of the recombination and diffusion terms in the presence of thunderstorm electric fields. We expect that the process of ionic recombination is not much influenced by the electric fields since ionic velocities of thermal agitation are much larger than the drift velocities in electric fields. For example, in a field of 1 stat. v./cm. the thermal velocity exceeds the drift velocity by a factor of nearly 100. The rate of loss by ionic diffusion is not wholly independent. From the energy standpoint, no, or few, ions of a given sign can reach the drop surface when the thermal energy is less than the electrical energy when Q is repulsive. In the absence of extraneous electric fields, the cutoff for the diffusion loss of a given sign ion therefore occurs when the repulsive drop charge equals $3akT/2e$, where k is the Boltzmann constant and T is the temperature. If the net drop charge is taken as equal to the polarization charge $(3/4)Ea^2$, following to a first approximation the probability of induction charge transfer between colliding drops [9, 10], then cutoff occurs for $E = 2kT/ea$. If the diffusion and conduction terms in (3) are compared for this same net drop charge, we find that in the presence of the electric fields the diffusion current is equal the conduction current when $E = 2.37D/ua$, which, upon substituting the relation $D/u = kT/e$ from kinetic theory, reduces to $E = 2.37kT/ea$. Using $T = 273^\circ \text{ K.}$ gives $E = 2.18 \times 10^{-4}/a$. For larger values of E the loss of ions by conduction exceeds the diffusion loss. These results suggest that the diffusional ion loss is important for small values of the electric field and small droplet size. When the electric fields are appreciable (i.e., of the order of 1 e.s.u. or greater), then the rate of ion loss by diffusion is expected to be much smaller than the ion loss by conduction. On this basis, we can assume the independence of the separate terms to be valid; that is, the diffusion term is appreciable when E is small and negligible when E is large, so an assumption of independence does not alter our result.

Equilibrium of the ionic concentration is established when the rates of ion production and ion loss are equal. Within the cloud body, the ion loss from the divergence flow is negligible, whereby at equilibrium

$$\frac{dn_1}{dt} = 0 = q - \alpha n_1 n_2 - 4\pi a D N n_1 - \frac{\pi n_1 u N}{3Ea^2} [3Ea^2 - Q]^2 \quad (4a)$$

$$\frac{dn_2}{dt} = 0 = q - \alpha n_1 n_2 - 4\pi a D N n_2 - \frac{\pi n_2 u N}{3Ea^2} [3Ea^2 + Q]^2. \quad (4b)$$

As a first approximation for the central cloud body, we assume that the drop net charge is negligible, whereby the equations are symmetric in n_1 and n_2 . It is useful to express our results in terms of the liquid water content of the cloud, $W = (4/3)\pi N a^3 \rho_w$, where ρ_w is the drop density, in which case (4) becomes

$$\frac{dn}{dt} = 0 = q - \alpha n^2 - \frac{3Wn}{\rho_w} \left(\frac{D}{a^2} + \frac{3Eu}{4a} \right) \quad (5)$$

with the solution

$$n = -\beta + \beta(1 + q/\alpha\beta^2)^{1/2}, \quad (6)$$

where

$$\beta = \frac{3W}{2\alpha\rho_w} \left(\frac{D}{a^2} + \frac{3uE}{4a} \right).$$

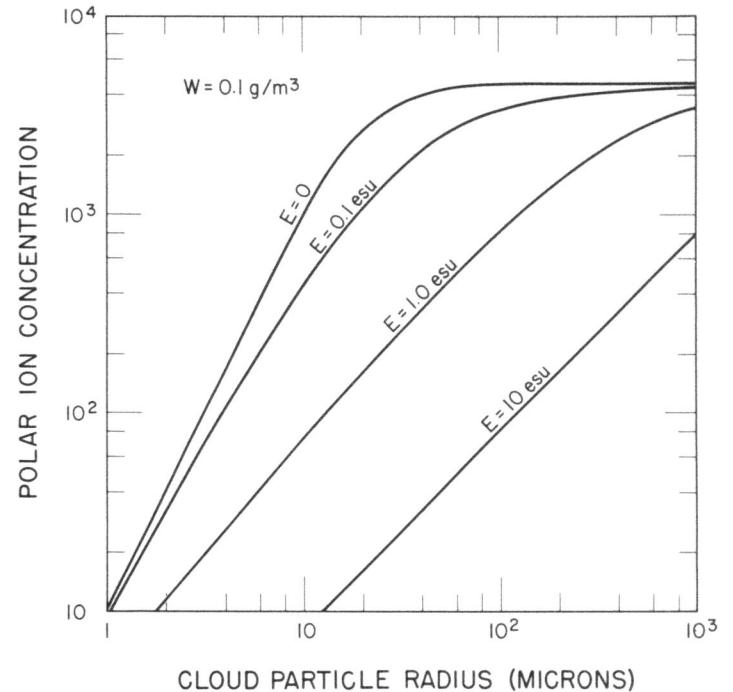


FIGURE 1.—Small ion concentration variation with cloud particle radius and electric field intensity for a cloud with liquid water content $W = 0.1 \text{ gm./m.}^3$

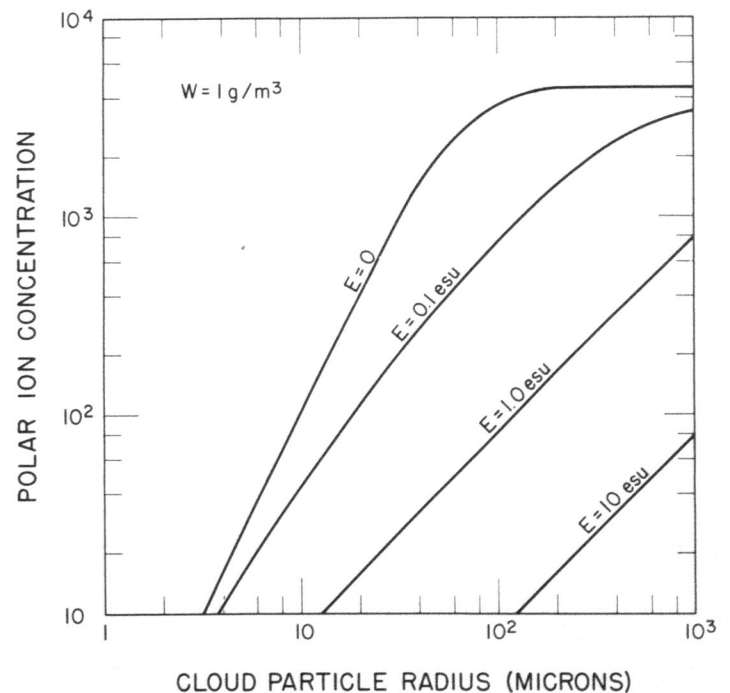


FIGURE 2.—As figure 1, but for a cloud with water content $W = 1.0 \text{ gm./m.}^3$

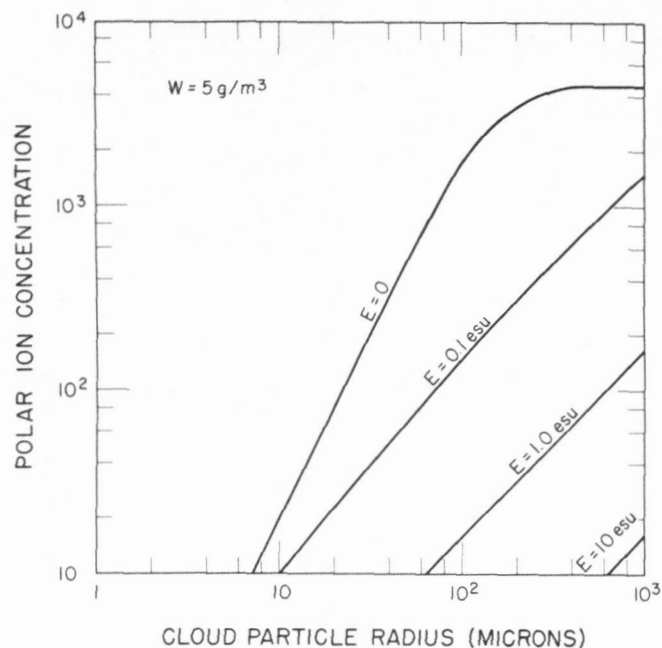


FIGURE 3.—As figure 1, but for a cloud with water content $W=5.0$ gm./m.³

Using $D=10^{-1}$ c.g.s., $\alpha=1.5 \times 10^{-6}$ c.g.s., $u=1.65 \times 10^3$ cm.²/sec. stat. v. and $q=30$ ions cm.⁻³ sec.⁻¹ commensurate with a 9-km. cloud level, we have plotted the equilibrium ion density as a function of drop radius in figures 1, 2, and 3 for cloud liquid water contents of 0.1 gm./m.³, 1.0 gm./m.³, and 5.0 gm./m.³

From these figures it is clear that at low values of the electric field, the ion concentration in clouds of small droplet radii is small as a result of the diffusion current to the large surface area of the droplets. As the droplet radius increases, the surface area of the droplets for a given water content is reduced and the ion loss by the diffusion mechanism decreases. At large droplet radii and small fields the diffusion current is negligible and the controlling mechanism is ionic recombination. For cloud volumes having electric fields in excess of 0.1 stat. v./cm. the primary loss occurs by ionic conduction to the polarized drops. A large reduction in the ionic concentration occurs in clouds subjected to electric fields of even moderate strength. For example, in clouds of 1 gm./m.³ water content having average droplet radii in the range from the smallest droplets to 100 μ , the ionic concentration in the presence of a 1-stat. v./cm. (300-v./cm.) field is decreased by a factor of nearly 100 from that which would occur by diffusion and recombination losses alone in the absence of the field. Fields of 10 stat. v./cm. decrease the ion concentration to almost negligible values for all cloud drop size distributions normally encountered in clouds.

The values of the ion concentrations have been computed on the basis of negligible net droplet charge. The consequences that follow when the cloud volume has a net charge carried on the drops will be considered later. We may note here that if the induction charging mechanism is present, providing equal numbers of positive and negative drops, the resulting ion concentrations are essen-

tially unchanged from those that exist when the cloud is composed of uncharged drops. This result is evident from the recent findings by Sartor [10] that when two drops or spherical particles of equal radii touch in line with an existing electric field, the charge transferred is given by $Q=1.65Ea^2$. If this charge is included in the terms of (1) relating the conduction loss of ions to the droplet charge, then the result is obtained that the respective ion currents to half the droplets are approximately doubled and approximately halved to the remaining droplets. Since the colliding droplets usually will not be aligned with the field the charge transferred by induction and the excess ion loss occurring as a result of the conduction mechanism will be somewhat less than this estimate. Thus we may expect that the ion concentrations in the presence of inductively charged drops are somewhat less, but not greatly so, than those within clouds where $Q=0$. Note particularly that this result holds in the absence of selective separation of the inductively charged drops into finite cloud volumes.

3. IONIC EQUILIBRIUM WITHIN THE CLOUD BOUNDARY

The conditions for ion equilibrium in the boundary layers of thunderclouds require close scrutiny. The radial component of the electric field at the cloud boundary originating from the primary positive dipole charge distribution of the thundercloud causes the creation of a net space charge shielding layer at the boundary [4, 5]. As a result of the reduced ion concentration within the central cloud, the flow of that sign ion repulsed outwardly from the central cloud into the boundary layers by the action of the field is small. Outside the cloud surface the concentration of that sign ion attracted toward the cloud is equal to or greater than that representative of the free air at the same level; we should expect it to be somewhat greater in fact, since the lower concentration of the repelled ion of opposite sign in the electrode region immediately surrounding the cloud-air interface causes the recombination ion loss to be less than the rate of ionization. The flow of that sign ion attracted inwardly from the free air surrounding the cloud is therefore large. As a result the ratio of the polar ion concentrations is greatly different from unity in the cloud boundary layer and the droplets systematically acquire that sign charge determined by the larger ion concentration. The droplet charge at equilibrium is given by (2), which represents the result that at equilibrium the polar conduction currents to the drop are equal.

The argument for determining the ion concentrations in the boundary layers is thereby altered from that within the central cloud, because $n_1 \neq n_2$ and because the cloud particles in the boundary layer are unipolarly charged. The analysis here is made only semi-quantitatively.

Examination of figures 1-3 shows that in clouds the ion concentration remains low even for low water contents and low fields. For example, if in the upper cloud boundary the radial outward-driven electric field is only 1 stat.

v./cm., while the liquid (solid) water of 0.1 gm./m.³ is distributed as 100- μ particles, then the ion concentration in this region if the droplets were uncharged would be only about one-sixth that which would exist at the same level in the absence of the cloud. If we now recognize that the cloud particles are *systematically* charged by the hyper-electrification mechanism, a more accurate estimate of the concentration can be made by a successive approximation. Using the factor of one-sixth for the ratio of n_1/n_2 , we obtain by (2) an average particle charge of $1.25 Ea^2$. With this value of particle charge the rate of loss of the outward-driven positive ion is approximately double what the rate would be if the particles were uncharged. Thus the factor of reduction of the outward-drive ion should be more nearly one-twelfth the free-air value at the same level. This argues that the concentration of the ion repelled from the central charge of thunderstorm clouds remains low until relatively close to the cloud-air interface where the shielding charge layer has effected an appreciable field reduction and where turbulent mixing processes exchange ions and dry air across the cloud-air boundary.

In the boundary layer within the cloud, the outward-driven ion concentration is decreased toward the cloud boundary because the recombination product $\alpha n_1 n_2$ increases outwardly and because the conduction current term, of (4), increases since the cloud droplets are charged oppositely in the sign of the incoming higher concentration ion. This picture is altered only as the cloud thins sufficiently and as penetration outward through the shielding charge layer results in a radial electric field sufficiently reduced to make recombination the primary loss mechanism. Then $q > \alpha n_1 n_2$ and the concentration of the outward-driven ions increases.

The concentration within the cloud boundary layer of the ion attracted into the boundary from the free-air environment is deduced similarly, but here the argument is altered by two important factors: (1) the flow of ions into the cloud boundary layer is large because of the relatively large ion concentration outside the cloud-air surface; and (2) the cloud particles are charged in the same sign as the attracted ion.

Examining the separate mechanisms of ion loss for the inflowing ion, we find that the rate of loss from recombination in the boundary layer will be small as a result of the low concentration of ions of the opposite sign. The diffusional ion loss will also be small if appreciable electric fields exist within the cloud boundary, as can be seen from our previous argument. If in the boundary region the attracted ion concentration is six times the outward-flowing ion concentration, the hyper-electrification charge on the cloud particles is $Q = 1.25 Ea^2$. Table 1 shows the expected drop charge in the boundary layer for electric fields of 0.1, 1.0, and 10 stat. v./cm. together with the diffusion cutoff charge $Q_c = 3akT/2e$ (for $T = 273^\circ$ K.) determined on the basis that an ion cannot reach the surface of the cloud droplet when the repulsive energy is greater than the thermal energy (again Q_c is for $E = 0$, where E is the external field in the absence of the drop).

TABLE 1.—Expected drop change Q in the boundary layer for electric fields E of 0.1, 1.0, and 10 stat. v./cm., and diffusion cutoff charge Q_c

Drop radius (μ)	Q Hyper-electrification			Diffusion cutoff charge, Q_c
	$E=0.1$	$E=1.0$	$E=10$	
1	1.25×10^{-9}	1.25×10^{-8}	1.25×10^{-7}	1.18×10^{-8}
10	1.25×10^{-7}	1.25×10^{-6}	1.25×10^{-5}	1.18×10^{-7}
20	5×10^{-7}	5×10^{-6}	5×10^{-5}	2.36×10^{-7}
40	2×10^{-6}	2×10^{-5}	2×10^{-4}	4.72×10^{-7}
100	1.25×10^{-5}	1.25×10^{-4}	1.25×10^{-3}	1.18×10^{-6}
200	5×10^{-5}	5×10^{-4}	5×10^{-3}	2.36×10^{-6}
1000	1.25×10^{-3}	1.25×10^{-2}	1.25×10^{-1}	1.18×10^{-5}

These results show that the loss of the attracted ion by the diffusion mechanism is important only for the combination of cloud droplet distributions of radius less than 20 μ and fields of less than 1 stat. v./cm. This conclusion is valid for, say, all $n_1 < 3n_2$ since the charge acquired in the boundary layer is not critically sensitive to the estimated ratio of the concentrations.

Finally, the rate of loss of the inflowing ion by the conduction mechanism is reduced by the presence of the charge of like sign on the cloud particles. Using the estimate that within the upper cloud boundary $n_2 = 6n_1$, we obtain for the bracketed quantity in equation (4b)

$$[3Ea^2 - 1.25Ea^2] = [1.75Ea^2]$$

whereby the ion loss within the shielding layer is diminished to nearly 1/3d that which would occur within uncharged clouds. (The factor of reduction of the negative inflowing ion from the free-air value is thus 1/2 and since the outflowing reduction was 1/12, we find that our estimate $n_2 = 6n_1$ has been mutually consistent.)

Thus the ion loss for the inflowing ion at and within the cloud boundary layer from recombination and diffusion to the cloud particles is negligible, while the loss by conduction to the drops is reduced to about one-third that which would occur in the absence of the shielding layer charge distribution in the boundary region. With these conditions associated with ion flow from outside the cloud-air boundary taken into account, we see that the concentration of the attracted ion at the cloud boundary is large and tends to remain large with increasing penetration into the cloud boundary sheathing charge layer.

4. THICKNESS OF THE SHIELDING LAYER CHARGE DISTRIBUTIONS

Within the cloud boundary the cloud particles are charged by the initial asymmetry of current flow to the polarization surface charge distribution on the individual particles as outlined above. The principal ion loss mechanism in the presence of thunderstorm electric fields is by conduction. Under an initial assumption that the particles in the boundary layer are uncharged, the conduction loss of the inflowing ion current is

$$\frac{dn}{dt} = -3\pi a^2 E u N n \quad (7)$$

and the time required for reducing the ion concentration by 87 percent of its value at the cloud-air interface is

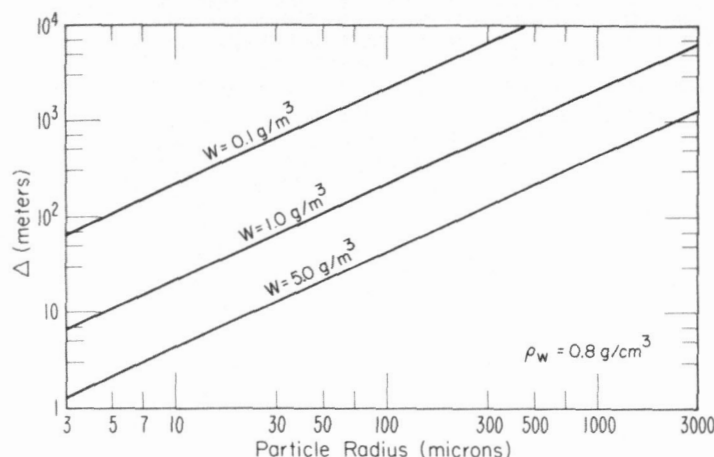


FIGURE 4.—Thickness of shielding charge layer along static cloud boundaries as a function of particle radius for clouds with water content $W=0.1$ gm./m.³, $W=1.0$ gm./m.³, and $W=5.0$ gm./m.³ ($\rho_w=0.8$ gm./cm.³)

$2\tau = 2/3\pi a^2 Nu E$. The charge captured from the current in the initial period of 2τ sec. is small compared with the total charge required to establish the shielding charge distribution. Successive incoming ions are in part repulsed from capture by the accumulating charges on droplets near the cloud boundary and are swept farther inward under the action of the field prior to capture by less well charged drops. As we have estimated in the preceding section, the ion loss that occurs at equilibrium within charged boundary layers is reduced to approximately one-third that which occurs within uncharged droplet distributions. On the basis of this estimate, the shielding layer charge distribution thickness is such that 87 percent of the inflowing ions are captured in a distance $\Delta = 3Eu(2\tau)$ or

$$\Delta = \frac{2}{\pi Na^2} = \frac{8a\rho_w}{3W}, \quad (8)$$

where Eu is the ionic velocity within the boundary layer. This result is plotted in figure 4 for clouds with a water content of 0.1 gm./m.³, 1.0 gm./m.³, and 5.0 gm./m.³. These curves infer the thickness of the boundary charge distribution in the absence of convective charge transport. The linear curves are plotted assuming $\rho_w = 0.8$ gm./m.³.

At the base of the cloud the droplet concentration in continental clouds may approximate $200/\text{cm}^3$. For an average droplet diameter of 4μ the liquid water content is close to 0.1 gm./m.³ and the ion penetration distance is about 100 m. The shielding layer thickness is extended continuously by the convective motions of the cloud and in a minor way by the conduction transport of charged drops.

At the cloud top the structure of the cloud together with the role of the precipitation within the water storage volume in the cloud updraft must be considered. As outlined in a succeeding paper [7], the development of the precipitation regime acts to reduce the vertical motion and the water content in the uppermost cloud levels. The particle density is greatly reduced as a result of the decreased water content and (especially when we recognize that the ice phase is present) the relatively large particle

size associated with precipitation development. Under these circumstances, the upper cloud becomes transparent to the flow of small ions from outside the cloud. For example, if the upper cloud water content is 0.1 gm./cm.³ and the particle size is 100μ then the sheathing layer thickness exceeds 2 km. Over the central convective updraft regions the cloud density will normally increase with decreasing elevation in the cloud top and the flow of ions will be arrested within more shallow charge layers. This is important to the convective transport of charge within storms.

5. EFFECT OF ICE PHASE IN CLOUD

So far, our discussion has been on the basis of a cloud of spherical water droplets that behave, as regards the ionic capture processes, as conducting spheres. The principal loss of ions in clouds having electric fields occurs from the conduction currents to the polarization and net free charges carried on the cloud drops. This loss is continuous even at drop charge equilibrium when the positive and negative currents to the drop are equal. The neutralizing of the polar currents is by internal conduction within the drop.

The equivalence of the conducting sphere and the water drop is completely valid for the present discussion. The potential function for a dielectric sphere is

$$V = -Ea \cos \theta \left(1 - \frac{(\epsilon - 1)a^3}{(\epsilon + 2)r^3} \right), \quad (9)$$

where ϵ is the dielectric constant. For water, ϵ is near 80 and hence the factor $(\epsilon - 1)/(\epsilon + 2)$ approaches unity and the potential function reduces to that of a conducting sphere. The conductivity of rain water is of the order of 10^{-4} (ohm cm.)⁻¹ [1], implying that the internal resistance of the drop and the relaxation time for charge transfer within it is negligibly small (of the order of 10^{-7} sec.) compared with the charging time of the drop and the resistivity of the air outside it.

For ice in a static electric field, the dielectric constant again is near 80 ; the conductivity, however, is reduced by a factor of 10^4 or 10^5 [1]. Thus the potential equation is unchanged when an ice sphere is substituted for the drop, while the relaxation time increases to the order of 10^{-2} sec. From this it is evident that an ice sphere is entirely equivalent to an equal-sized water drop as regards the ionic conduction processes within the cloud. The argument need not be changed to apply to other than spherical particles because we expect the primary particles in the cloud tops to be prismatic crystals and snow pellets, both of which approach spherical geometry.

It is commonly believed that the ice phase is important to the thunderstorm mechanism. In the present concept it is clear that the effect of ice nucleation and cloud glaciation first of all is to facilitate the precipitation mechanism. The number of ice particles is few relative to the number of drops, and the processes associated with glaciation rapidly and greatly diminish the upper cloud particle density. The result is that the thickness of the boundary

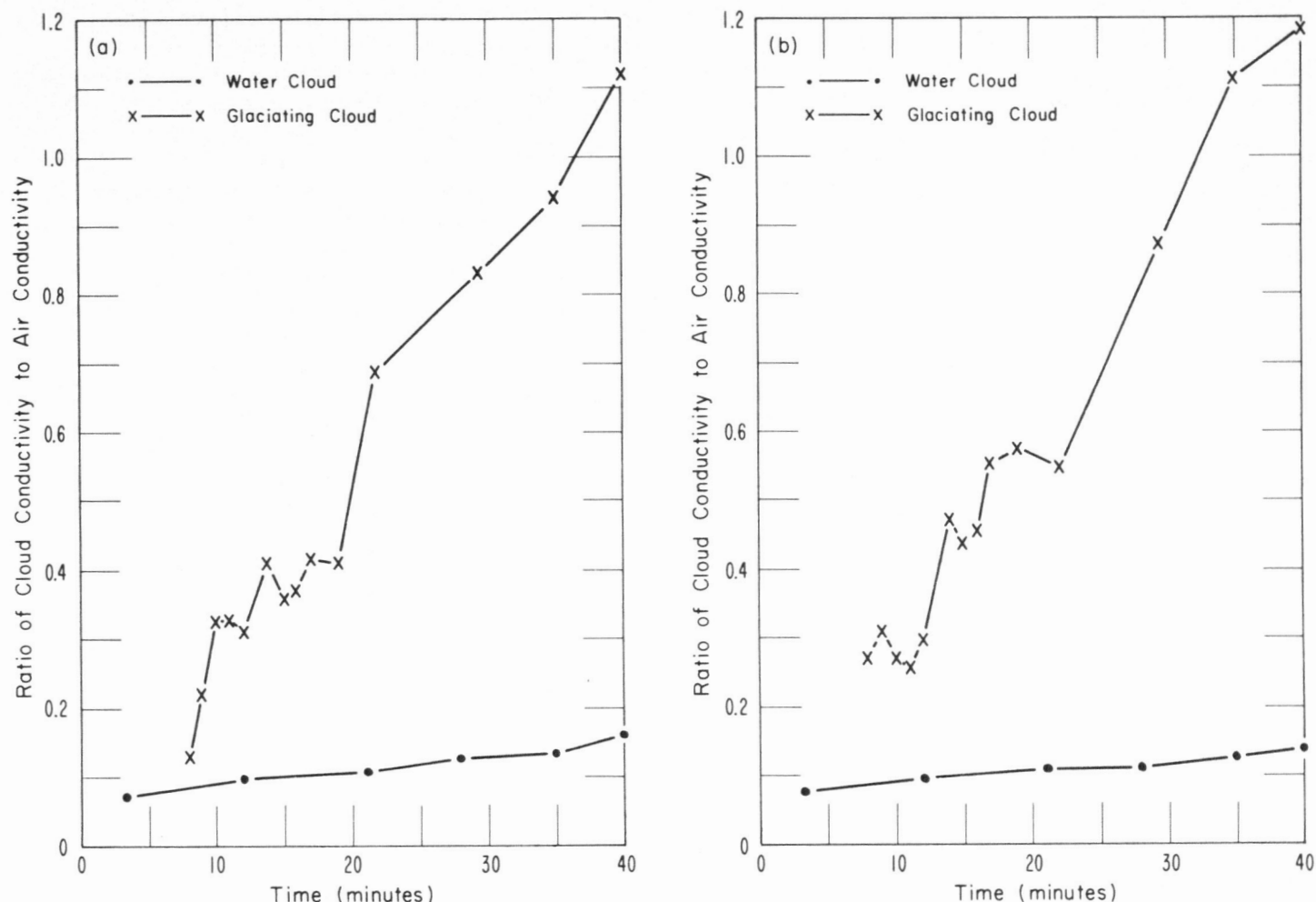


FIGURE 5.—(a) Time variation of the ratio of the positive conductivities before and after cloud formation for a pure water cloud and a glaciating cloud. Electric field $E=0$. (b) Same as (a) except for negative conductivities.

layer charge distribution and the concentration and flow of the attracted ion to greater depths within the upper cloud are increased. A second result may also be important. For an individual cloud particle an almost immediate effect of glaciation is a reduction in the density of the particle by a factor of from 4/5 to 1/10. We may expect that such particles are thrust upward into the upper levels of the cloud where the polar ion concentrations are greatly different and where conduction charging processes are most active. Because of such exposure and the relatively larger diameter of the ice particles, the electrification by conduction within the upper shielding region may occur disproportionately on particles capable of growing by the Bergeron and accretion mechanism in combination. These factors together suggest the glaciation process may appear to "trigger" the flow of charge to the precipitation.

Measurements have not been made of the increase in ion concentration during the glaciation of natural clouds. The usual method of conductivity and ion measurement with the Gerdien-type apparatus is not feasible because of triboelectric effects. Reliable measurements of the conductivity during cloud glaciation were made some years ago in a 3000-m.³ cloud chamber. No electric fields were present. The results of these measurements are shown in

figure 5. The continuous curve at the bottom of each plot indicates the ratio of the polar conductivity in the cloud to the conductivity prior to cloud formation as a function of time for a supercooled water cloud formed by an expansion of moist air from 2.75 lb./in.² gage pressure. Such clouds persisted for about 1 hr. within the cloud chamber. The initial liquid water content immediately after expansion was 1.25 gm./m.³ On a succeeding day a similar expansion was made from 3.75 lb./in.² gage pressure; the initial liquid water content was 1.71 gm./m.³ A few minutes following cloud formation the supercooled cloud (-10° C.) was seeded heavily with dry ice. The \times -points in figure 5 show the increase with time of the ratio of the conductivities in the ice crystal cloud to the conductivities prior to cloud formation. The slope of the points is in part the result of crystal precipitation from the chamber. Notes taken during the measurement indicate that a "good ice crystal cloud" remained 22 min. after cloud formation.

6. CLOUD DROP CHARGE AND THE ELECTRICAL CONDUCTIVITY CAUSED BY CHARGED CLOUD DROPLETS IN STATIC BOUNDARY LAYERS

The motion of the charged droplets under the action of the electric field contribute to the conduction currents

within the cloud. The conductivity, λ_d , arising from the cloud droplet current is given by $\lambda_d = NQu_d$, where u_d represents the migration velocity of the droplet in unit field, i.e., the droplet mobility. Equating from Stokes law gives for the droplet mobility $u_d = Q/6\pi a\eta$, where η is the viscosity of the air in the vicinity of the droplet. Thus the conductivity arising because of the charged droplet distribution is given by

$$\lambda_d = \frac{NQ^2}{6\pi a\eta} \quad (10)$$

The current flow inward from the sheathing layer cannot be greater than the flow to the cloud boundary. Remembering that the conductivity inside the cloud is the sum of the small ion and cloud drop conductivities shows $\lambda_0 E_0 \geq \lambda_d E$, where the left hand side represents the current density in the air outside the cloud and the right hand member is the current carried by the droplets. Using (10) gives the drop charge at the inner surface of the shielding charge as

$$Q \leq \left[\frac{6\pi a\eta E_0 \lambda_0}{NE} \right]^{1/2} \quad (11)$$

For a numerical example consider a quasi-static boundary along the cloud base where the field outside the cloud is 1 stat. v./cm. and within the boundary layer is as much as 15 stat. v./cm. Taking $a = 5 \mu$, $\eta = 1.7 \times 10^{-4}$ c.g.s., $\lambda_0 = 2 \times 10^{-3}$ e.s.u., and $N = 200 \text{ cm.}^{-3}$ gives $Q = 10^{-6}$ e.s.u. = 2000e. The velocity, $v = u_d E$, of the drop inside the boundary layer where the field is 15 stat. v./cm. is approximately 10 cm./sec. From this it is seen that the motion from electric forces alone of drops charged in the boundary layers is small compared to convective cloud motions.

7. CLOUD DROPLET CHARGE AND SMALL ION CONDUCTIVITY IN THE CUMULUS UPDRAFT

At the base of the cloud the convective removal of charged droplets upward through the boundary layer charging region limits both the accumulation of charge at the boundary and the charge deposited on the individual droplets. The radial electric fields at and outside the cloud-air interface may be large and consequently the ion flow to the cloud surface will increase. Despite this increase the competition for ions by the cloud drops is sufficiently great that the droplets receive but a small fraction of the charge that would be acquired at the cloud boundary region in the absence of convective transport. This we may show more clearly by the following example. If the updraft velocity V through the cloud base is 5 m./sec., then the number of droplets originating and passing upward is $NV = (200)(500) = 10^5 \text{ cm.}^{-2} \text{ sec.}^{-1}$. In the absence of a fully developed boundary charge layer, the radial inwardly directed field at the cloud surface (which results from the nearby lower negative charge of the positive primary thunderstorm dipole) may be $E = 10$ e.s.u. and the number of ions reaching the cloud-air interface of the order of $Eun_1 = (10)(6 \times 10^2)(1.5 \times 10^8) = 9 \times 10^6 \text{ cm.}^{-2} \text{ sec.}^{-1}$. The individual drop charge is

therefore only 90 e/drop and the total space charge density carried by the drops is $+18000 \text{ e/cm.}^3$

In the convective updraft within the central cloud volume the individual droplet charge remains almost constant and the small ion densities remain near the reduced level that would exist if the droplets were uncharged. This result follows from the charge limitation occurring during either strong or weak convective transport within the boundary layer at the cloud base. For strong convection the charge is limited by the competition for ions by the droplets. For weak convection the charge limitation follows because the inflowing ions are captured near the cloud-air interface, whereas the regions of strongest electric fields are displaced inward by the cloud flow. In the updraft the droplet size and the electric field increase while ions of either sign are added in equal numbers by the continuing ionization within the updraft. The negative ions are rapidly deposited by conduction on the positively charged droplet distribution. The positive ions are also rapidly deposited since assuming minimum values with the cloud of $E = 1$ e.s.u. and $a = 5 \mu$ gives, for a unipolar ion concentration, the equilibrium hyperelectrification charge per drop of $Q \sim 7.5 \times 10^{-7} \sim +1500e$. This minimum value charge is much larger than the probable drop charge of 90e. Thus the small ionic conductivities are low in the updraft while the positive charge per drop remains constant with height above the cloud base.

8. LIFETIME OF IONS IN CLOUDS

The mean life of the small positive ions within the cloud volume is given by

$$\Theta_1 = n_1/q = 1 / \left(\alpha n_2 + 4\pi aDN + \frac{\pi uN}{3Ea^2} [3Ea^2 - Q]^2 \right) \quad (12)$$

and a similar expression for n_2 . In the body of the cloud, where fields are normally equal or greater than 1 e.s.u., the primary loss mechanism is by conduction and the expression above for the case of uncharged droplets is approximated by

$$\Theta = 1/3\pi a^2 E u N. \quad (13)$$

Evaluation of this equation for reasonable values of the variables shows that the mean life of the small ion within the cloud is 1 sec. or less. When the particle density and the electric field are both reduced, as for example in the upper cloud boundary region, the rate of ion loss by conduction is small and the primary loss occurs by recombination. From the full equation (12) the mean life of the ion is near 80 sec. in clouds where $W = 0.1 \text{ gm./m.}^3$ and $E = 0.2$ e.s.u. Note that in (12) the assumption that Q is small compared with $3Ea^2$ follows when the recombination loss is dominant.

9. IONIC CONDUCTIVITY IN THUNDERCLOUDS

The estimated ratio of the ionic conductivity in cloud to the free-air conductivity at the same elevation, λ_c/λ_a ,

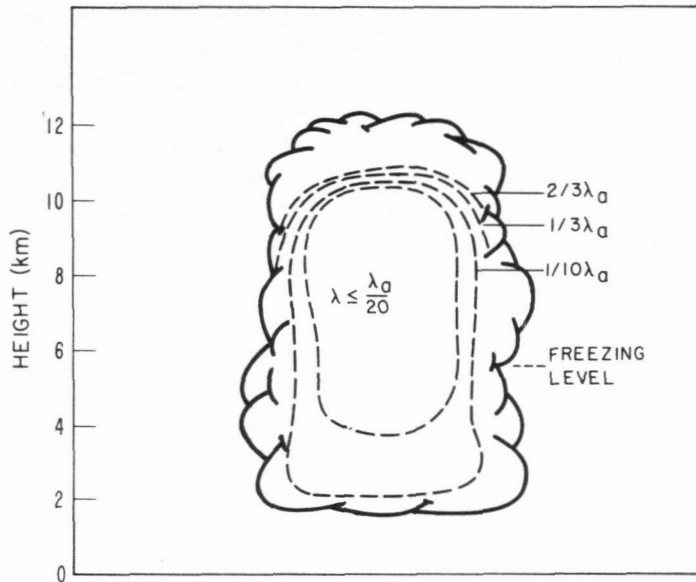


FIGURE 6.—Estimated total conductivity in thunderclouds expressed as a fraction of the free-air conductivity λ_a at the same height.

is shown in figure 6. At the cloud base and along the lateral cloud walls the conductivity decreases rapidly with cloud penetration as a result of ion transport to the large surface area of the small-sized droplet distribution. The central cloud volume is a region of maximum droplet surface area and here the polar conductivities are reduced to very low values, principally as a result of the conduction currents to the cloud droplets under the action of the electric fields within the cloud. Toward the upper cloud surface the precipitation mechanisms act to reduce the surface area of the cloud particulate distribution, resulting in an increase in the cloud conductivity toward the free-air value throughout an appreciable depth within the cloud boundary.

Figure 7 shows the ratio of the positive and negative conductivities in the cloud system. The central cloud, where the conductivities are each reduced by a large factor, is a region of approximate equality of the two conductivities. If there is any difference in the polar values, the ionic concentrations are so low as to render it inconsequential. The conductivity ratio differs greatly from unity near the regions of large gradient in the conductivity in figure 6 because both the total conductivity and the conductivity ratio are functions of the gradient of the cloud particle density. Immediately within the cloud base the positive conductivity exceeds the negative conductivity as a result of the positive current flow from the conducting environment toward the primary lower negative charge center of the thunderstorm dipole. As long as the electric field is directed toward the cloud base, negative ions enter the cloud only as a result of ion production except when the vertical air motion exceeds the downward migration velocity of the ion in the field. The ion mobility at the level of the cloud base approximates $600 \text{ cm}^2 (\text{stat. v.})^{-1} \text{ sec.}^{-1}$ from which we see that vertical fields of 1 e.s.u. at the cloud base drive the negative ion

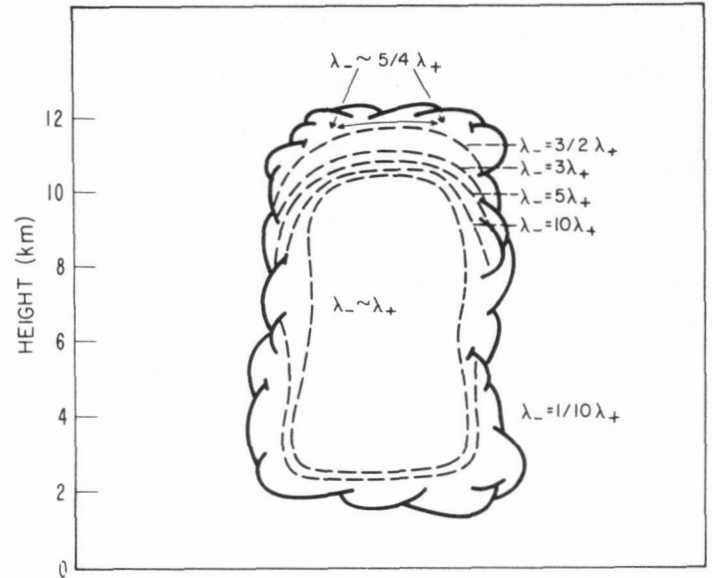


FIGURE 7.—Estimated ratio of the positive and negative conductivities in thunderclouds.

downward at 2 m./sec. even if the vertical air motion is 4 m./sec. Since strong vertical motion at the cloud base partly removes the shielding charge layer at the cloud boundary it seems clear that few negative ions reach the cloud base as long as the primary positive dipole of the cloud remains dominant in the charge distribution.

Near the upper cloud-air boundary the polar conductivities are not greatly different. This near-unity ratio of the conductivities at the cloud surface follows from the equation of ionic equilibrium. For dynamic current equilibrium the electric field at the boundary is reduced to about 0.2 stat. v./cm. as a result of the shielding charge layer within the cloud [6, 12]. At these field values the parameters involved in the equilibrium equations do not vary greatly during the average lifetime of the ion. Outside the upper cloud surface the concentration of negative ions is essentially unchanged from the free-air value of 4500 cm.^{-3} . Using this value together with $N=2 \times 10^{-2} \text{ cm.}^{-3}$, $\alpha=1.5 \times 10^{-6}$, $a=10^{-2} \text{ cm.}$ and $q=30/\text{cm.}^3$, we can solve the equilibrium equations by successive approximation for the ion concentration within the cloud boundary. By this method, the positive ion density near the upper cloud surface is found to approximate 2500 cm.^{-3} , whereas the negative ion density is reduced to near 3500 cm.^{-3} . Thus immediately within (a few hundred meters) the upper cloud surface the ratio of the conductivities approaches unity. Deeper within the shielding layer the particle density and the electric field increase. The equilibrium equations are no longer readily solvable for the ionic concentrations since the parameters of the equations vary markedly during the average lifetime of the ion. In this region the flow of negative ions downward greatly exceeds the outward flow of positive ions from the dense cloud-precipitation region below. The ratio of the negative to the positive conductivity thus increases with decreasing elevation until the negative ions are depleted by capture on the rising cloud particle current.

10. DISCUSSION

Equation (6) for ionic equilibrium in clouds was written in the form

$$n = -\beta + \beta(1 + q/\alpha\beta^2)^{1/2},$$

where

$$\beta = \frac{3W}{2\alpha\rho_w} \left[\frac{D}{a^2} + \frac{3uE}{4a} \right].$$

A few numerical computations are sufficient to show that $\beta^2 \gg q/\alpha$ for nearly the entire cloud system. (For example, for all clouds having $W \geq 0.1$ gm./m.³, $\bar{a} \leq 100$ μ , and $E \geq 1$ e.s.u.) The radical can therefore be expanded and the result written as

$$n = q/2\alpha\beta,$$

or as

$$n = \frac{q}{\pi N(4Da + 3uEa^2)}. \quad (14)$$

Again, for all $E \geq 0.1$ e.s.u. and $\bar{a} \geq 5$ μ the conduction term on the denominator is large compared to the diffusion term from which we have to a close approximation that in the body of the cloud

$$n = \frac{q}{3\pi NuEa^2}. \quad (15)$$

This is the result we have used in (13). It follows that the polar small ion conductivity is

$$\lambda = \frac{qe}{3\pi NEa^2} \quad (16)$$

and the ionic current density inside the cloud given by $i = E\lambda$ is

$$i = \frac{2qe}{3\pi Na^2} \quad (17)$$

which is independent of the field and constant for a given particle distribution. Thus it appears that the continuity of current flow argument which has been used previously [5, 6] for evaluating the general features of the space charge distribution in electrified clouds is not permissible on the basis of the small ion conductivities. This in no way negates the value of the previous charge distribution analysis but rather stresses that the charge distributions are limited by 1) breakdown and 2) convection. The shielding charge distributions arise by the same mechanism as outlined using the argument for current continuity, but the charging time will be less and the charge distribution magnitude will be greater in the absence of current continuity within the cloud. The charge centers within the cloud will accumulate charge at a rate close to $dQ/dt = I$, where I is the charging current to the charge centers, while the cloud as a whole remains nearly neutral as a result of the conduction flow of charge to the shielding charge distributions at the cloud boundaries.

The total conductivity together with the cloud particle velocity given by $v = Eu_d$, where $u_d = Q/6\pi\eta a$ is the particle

mobility, are important to the regeneration times following lightning discharge. The regeneration time for an observer distant from the storm is primarily the boundary layer adjustment in the upper cloud levels. This adjustment occurs in a time period controlled by the free-air conductivity at the level of the upper volume of the cloud and the recovery times will approximate 10 to 15 sec. for developed storms. For an observer near the storm the field recovery is the result of the superposition of all charge redistributions, including those occurring in the cloud base and the cloud top. These phenomena are essentially as described by Tamura [11]. The initial field recovery immediately following the lightning flash is believed primarily the result of boundary layer charging and discharging processes and as such may be influenced by increases in the lower air small ion conductivity caused by point discharge processes occurring at the surface of the earth below active storms.

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